



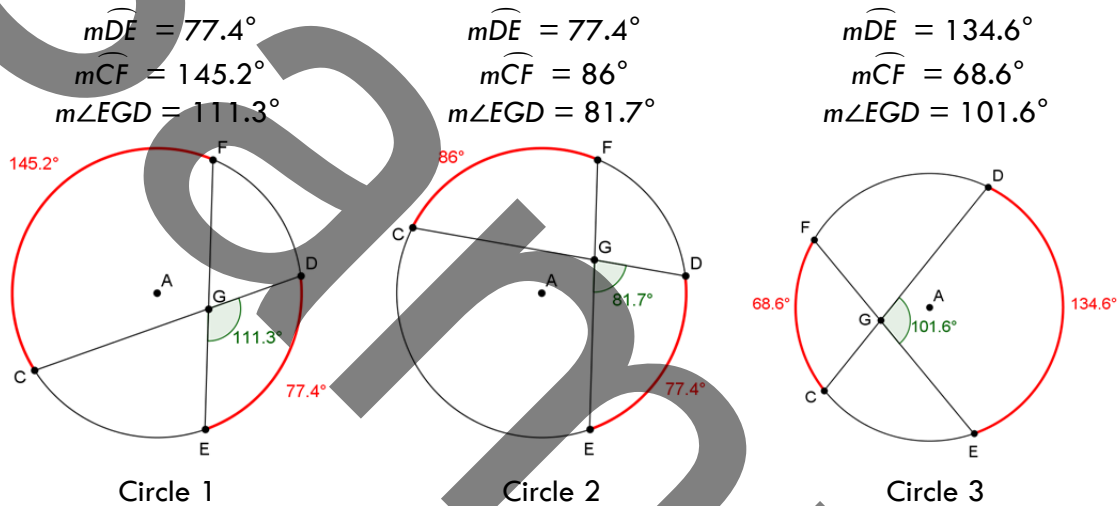
Angle Relationships in Circles

Elaborate – Answer Key

Directions: In each section below, compare the given information about arc measures or angle measures. Use your observations to complete the data table, and use the data table to answer the question that follows. Answer the debriefing questions.

Part 1: Angle formed by two intersecting chords

In circle A below, \overline{CD} and \overline{EF} are chords that intersect at point G. $\angle EGD$ intercepts \widehat{DE} , and its vertical angle, $\angle FGC$, intercepts \widehat{CF} .



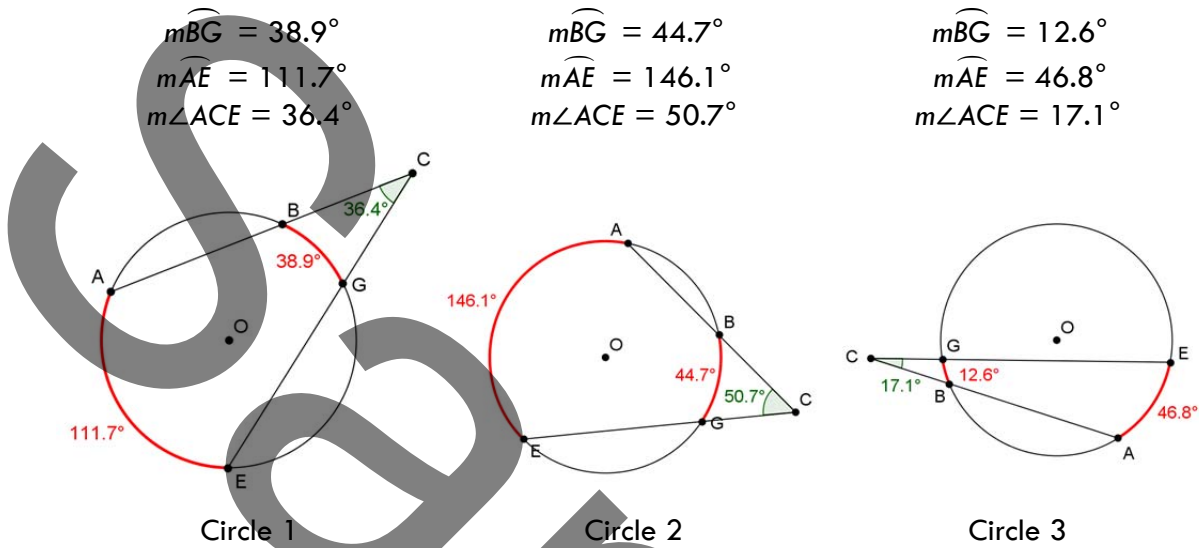
| Circle Number | 1 | 2 | 3 |
|---|--|------------------------------------|--|
| $m\widehat{DE}$ | 77.4° | 77.4° | 134.6° |
| $m\widehat{CF}$ | 145.2° | 86° | 68.6° |
| $\frac{m\widehat{DE} + m\widehat{CF}}{2}$ | $\frac{77.4 + 145.2}{2} = 113.3^\circ$ | $\frac{77.4 + 86}{2} = 81.7^\circ$ | $\frac{134.6 + 68.6}{2} = 101.6^\circ$ |
| $m\angle EGD$ | 113.3° | 81.7° | 101.6° |

1. What is the relationship between an angle formed by two chords and their intercepted arcs?

The measure of the angle formed by two intersecting chords is equal to one-half the sum of the measures of the intercepted arcs.

Part 2: Angle formed by two intersecting secants

In circle O below, \overline{AC} and \overline{EC} are segments of secants \overline{AC} and \overline{EC} that intersect at point C, outside the circle. $\angle ACE$ intercepts \widehat{BG} and \widehat{AE} .



| Circle Number | 1 | 2 | 3 |
|---|---------------------------------------|---------------------------------------|--------------------------------------|
| $m\widehat{BG}$ | 38.9° | 44.7° | 12.6° |
| $m\widehat{AE}$ | 111.7° | 146.1° | 46.8° |
| $\frac{m\widehat{AE} - m\widehat{BG}}{2}$ | $\frac{111.7 - 38.9}{2} = 36.4^\circ$ | $\frac{146.1 - 44.7}{2} = 50.7^\circ$ | $\frac{46.8 - 12.6}{2} = 17.1^\circ$ |
| $m\angle ACE$ | 36.4° | 50.7° | 17.1° |

2. What is the relationship between an angle formed by two secants and their intercepted arcs?

The measure of the angle between two secants that intersect outside the circle is equal to one-half the difference of the measures of the intercepted arcs.

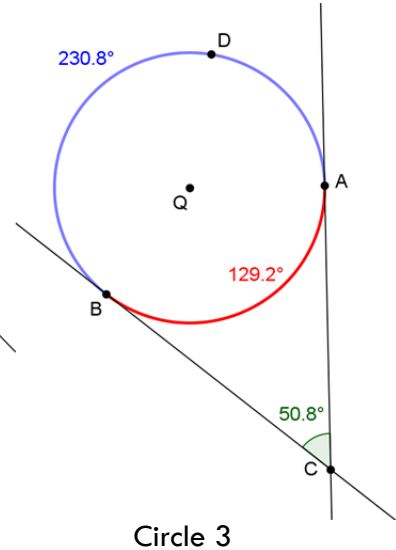
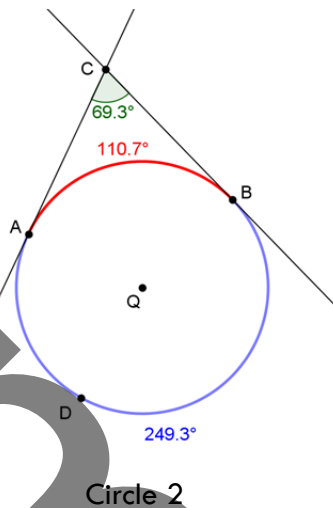
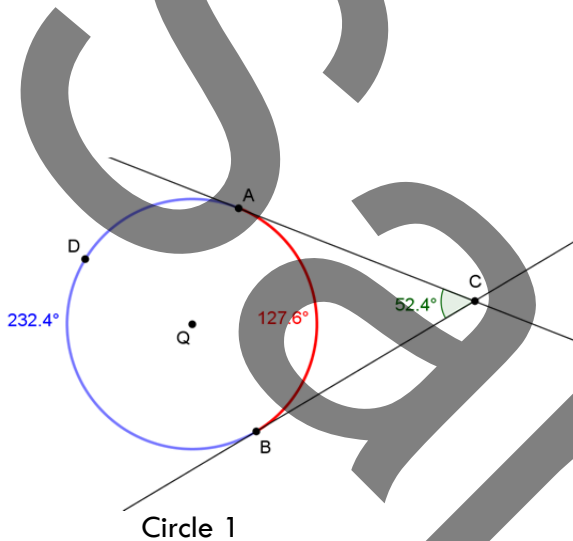
Part 3: Angle formed by two intersecting tangents

In circle Q below, \overline{AC} and \overline{BC} tangent lines that intersect at point C. $\angle ACB$ intercepts minor arc \widehat{AB} and major arc \widehat{ADB} .

$$\begin{aligned} m\widehat{AB} &= 127.6^\circ \\ m\widehat{ADB} &= 232.4^\circ \\ m\angle ACB &= 52.4^\circ \end{aligned}$$

$$\begin{aligned} m\widehat{AB} &= 110.7^\circ \\ m\widehat{ADB} &= 249.3^\circ \\ m\angle ACB &= 69.3^\circ \end{aligned}$$

$$\begin{aligned} m\widehat{AB} &= 129.2^\circ \\ m\widehat{ADB} &= 230.8^\circ \\ m\angle ACB &= 50.8^\circ \end{aligned}$$



| Circle Number | 1 | 2 | 3 |
|--|--|--|--|
| $m\widehat{AB}$ | 127.6° | 110.7° | 129.2° |
| $m\widehat{ADB}$ | 232.4° | 249.3° | 230.8° |
| $\frac{m\widehat{AB} - m\widehat{ADB}}{2}$ | $\frac{127.6 - 232.4}{2} = 52.4^\circ$ | $\frac{110.7 - 249.3}{2} = 69.3^\circ$ | $\frac{129.2 - 230.8}{2} = 50.8^\circ$ |
| $m\angle ACE$ | 52.4° | 69.3° | 50.8° |

3. What is the relationship between an angle formed by two tangents and their intercepted arcs?

The measure of the angle between two tangent lines to a circle is equal to one-half the difference between the measures of the intercepted arcs.

Debriefing Questions

1. How does the location of the intersection of the two lines or line segments relate to whether you add or subtract the measures of intercepted arcs to calculate the measure of the angle between the lines or line segments?

If the line segments (chords) intersect inside the circle, you will add the measures of the intercepted arcs before dividing by 2. If the lines or line segments (secants or tangents) intersect outside the circle, you will subtract the measures of the intercepted arcs before dividing by 2.

2. For two intersecting chords, there are 4 angles created around the point of intersection. If you know the measure of one of these angles, how could you determine the measures of the other 3 angles?

The sum of these angles will be 360° . Two adjacent angles form a linear pair, so they will be supplementary, and you can determine the measure of the second angle by subtracting the measure of the known angle from 180° . Two vertical angles will be congruent, so their angle measures will be equal.